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Experiments on magnetohydrodynamic co-rotation in mercury

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With 6 figures in the text

ABSTRACT

In an attempt to reproduce on laboratory scale the astrophysical phenomenon of co-rotation, experiments have been performed, which show in a qualitative way the influence of a strong magnetic field on the motion of a conducting liquid surrounding a rotating sphere.

General considerations

The astrophysical phenomenon of co-rotation arises from the fact that in the ionized interstellar gas surrounding a star having a general magnetic field the magnetic lines of force are effectively "frozen into" the matter, which is thereby forced to move in such a way, that in the stationary state the angular velocity is constant along any line of force (Ferraro, 1937). It can also be shown that, if a steady state exists, the magnetic field must be symmetric about the axis of rotation.

As in a laboratory-scale model experiment the dimensions and velocities are necessarily small and the conductivity limited, a state with frozen-in field lines can not be achieved. However, the influence of the magnetic field on the motion of the fluid can still be strong. The ratio between electromagnetic and viscous forces is of the order of magnitude of the number $B^2L_c^2\sigma/\eta$, where L_c is a characteristic dimension of the apparatus, B the magnetic field strength, σ the electrical conductivity and η the dynamic viscosity (Lehnert 1952, 1955; Lundquist, 1952). This number can be made much larger than unity even in a model experiment.

Experimental arrangements and results

In the model experiments reported in the present paper the ionized interstellar gas was represented by a mass of mercury contained between a copper sphere (radius $R = 62.5$ mm), rotating about a vertical axis, and a concentric outer stationary spherical vessel (radius $R_1 = 125$ mm) of copper. The angular velocity was kept small (about 0.2 s^{-1}) to avoid turbulence. For reasons of symmetry it was sufficient to fill the lower half of this volume, so that the liquid had a free surface in the equatorial plane (while the sphere was separated into two halves insulated from each other), see Fig. 1. Then the velocity distribution in this plane could readily be observed. The

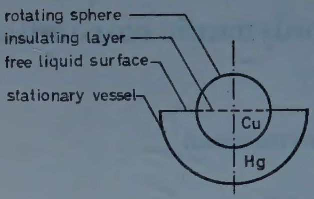

Fig. 1.

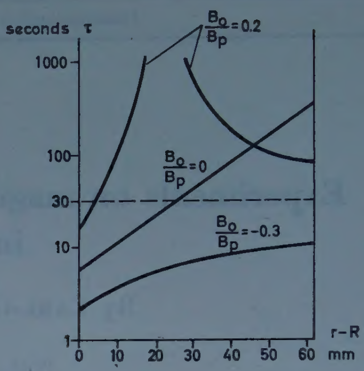

Fig. 2.

Fig. 1. Geometry of the experimental arrangement.

Fig. 2. Variation of $\tau = \varrho/\sigma B^2$ in the equatorial plane when $B_p = 960$ gauss.

arrangement had the disadvantage of introducing disturbing surface effects but had to be used, since other methods of velocity determination (such as probe measurements) were not feasible at the low velocities involved.

The magnetic fields used were superpositions of a dipole field and a homogeneous field:

$$\bar{B} = B_p\left(\frac{R^3}{r^3}\cos\vartheta,\ \frac{1}{2}\frac{R^3}{r^3}\sin\vartheta,\ 0\right) + B_0(\cos\vartheta,\ -\sin\vartheta,\ 0) \tag{1}$$

(spherical polar coordinates), where R is the radius of the rotating sphere. Actually, instead of a solid sphere, a spherical shell had to be used in order to give room for the source of the dipole field, but this arrangement could be estimated to have negligible influence on the phenomenon to be studied, provided the wall thickness was large enough, i.e., much larger than $R\,\sigma_{\mathrm{Hg}}/\sigma_{\mathrm{Cu}} \approx 0.02\,R$. Since strong magnetic fields were necessary to produce pronounced effects, the ohmic heating of the coils set a limit to the maximum duration, T, of each experiment. On the other hand, the time necessary to approach a stationary state can be estimated from the equations of motion to have the order of magnitude $\tau = \varrho/\sigma B^2$, where ϱ is the density of the liquid. Values of B_p used, were in the region of 1000 gauss, corresponding to a value of T of about 30 seconds, whereas the local values of $\varrho/\sigma B^2$ were those given in Fig. 2. Thus a stationary state could not be fully realized.

Although the surface film on the mercury was removed by means of a thin layer of nitric acid, surface effects tending to brake the motion were still serious and were especially aggravated by the inevitable contamination of the mercury by copper from the walls (copper being necessary to ensure sufficient electrical contact). Since the velocity distribution without magnetic field and with no surface effects can easily be calculated, a comparison with the measured values shows the surface influence in this case and gives a hint of its rôle in the cases when a magnetic field is present.

Fig. 3 shows photographs of the mercury surface on which grains of sand have been scattered. The path lengths of the grains during the exposure are proportional to the velocity of the mercury surface. The presence of the magnetic field is seen to introduce a greatly enhanced co-rotation.

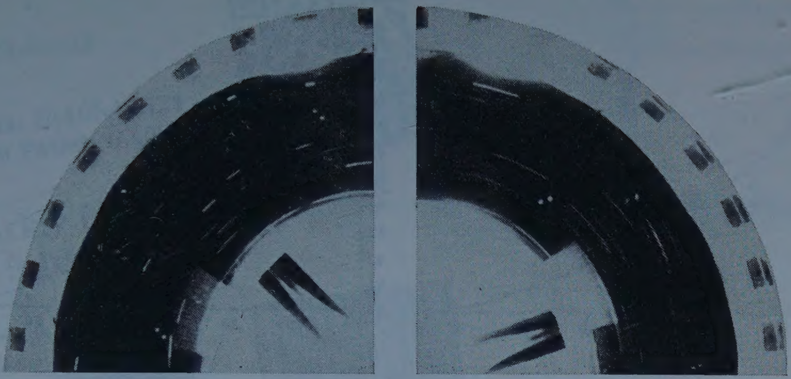

Fig. 3. Motion of mercury outside rotating copper sphere. To the left: Steady-state motion without magnetic field. To the right: State of motion when a magnetic dipole field ($B_p = 960$ gauss) has been present for 30 seconds.

Dipole field only ($B_0 = 0$)

Fig. 4 shows the velocity v_φ in the equatorial plane as a function of the distance, $r-R$, from the surface of the rotating sphere. The points plotted refer to a particular experiment, whereas the dashed lines indicate the region of variation of results from several experiments. An approximately steady state seems to prevail in an inner region, extending out to about 30 mm from the sphere, but not in the outer parts of the liquid. This observation is in agreement with what should be expected from Fig. 2.

The deviation of the measured velocity in the case of no magnetic field, from that calculated disregarding surface effects (solid lines in Figs. 4 to 6) is considerable. In the case of a strong magnetic field the same effects should be present and produce a similar deviation. An adequate quantitative correction for this disturbance cannot be given, but the results are not inconsistent with an almost rigid rotation (dotted line in Fig. 4) in the region out to about 15 mm from the sphere.

Measurements were also made in which the outer vessel was made of electrically insulating material. In these cases the acceleration of the liquid was slower (because the current system causing the acceleration could not close in the outer vessel), but there was still an inner steady-state region of about the same extension and with a velocity distribution lying within the dashed lines of Fig. 4.

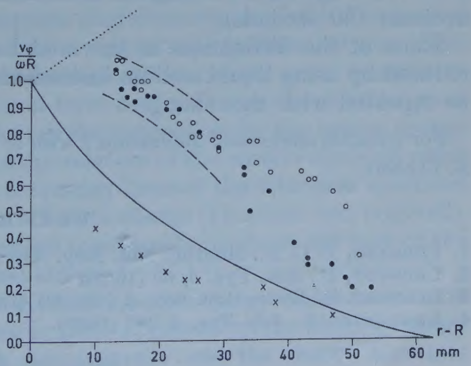

Fig. 4. Velocity distribution in the equatorial plane. Sphere and vessel made of copper. ($B_p = 960$ gauss. $B_0 = 0$.) × = No magnetic field; ● = Magnetic field present for 15 sec.; ○ = Magnetic field present for 30 sec.

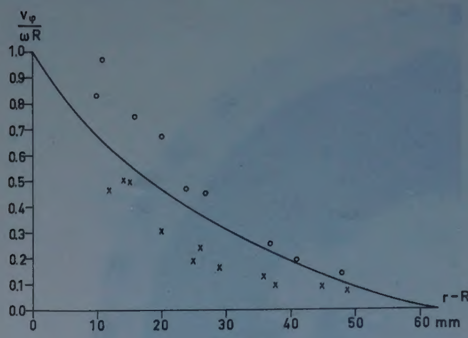

Fig. 5.

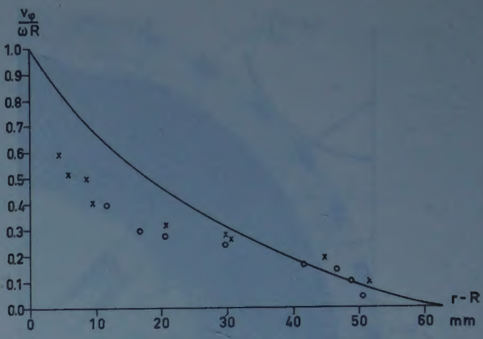

Fig. 6.

Fig. 5. Velocity distribution in the equatorial plane. Sphere and vessel made of copper. (B_p = 960 gauss, $B_0 = -300$ gauss.) × = No magnetic field; ○ = Magnetic field present for 30 sec.

Fig. 6. Velocity distribution in the equatorial plane. Sphere and vessel made of glass. ($B_p = 960$ gauss. $B_0 = 0$.) × = No magnetic field; ○ = Magnetic field present for 30 sec.

Dipole field plus homogeneous field

(*a*) Homogeneous field antiparallel to the magnetic moment ($B_0/B_p < 0$ in equation (1)): Fields of this type gave lower velocities than the pure dipole field. One case is given in Fig. 5. The results can be considered as an effect of the stronger coupling between the moving liquid and the outer wall caused by the addition of the homogenous field.

(*b*) Homogeneous field parallel to the magnetic moment ($B_0/B_p > 0$ in equation (1)): In this case no stationary state was attained in the time available (as is to be expected from Fig. 2). During the first 10–20 seconds the velocity increased rapidly in a region out to about the radius of zero field, $r_0 = R(B_p/2\,B_0)^{\frac{1}{3}}$, and decreased at larger radii (simply reflecting the action of the initial acceleration process), but afterwards the velocity distribution was deformed in an irregular way.

The magnetohydrodynamic coupling between the rotating sphere and the liquid depends on the electrical contact across the boundary between them. This could be demonstrated by using a sphere of insulating material. Fig. 6 shows results corresponding to those given in Fig. 4 but with the sphere (and the vessel) made of glass instead of copper. As is seen from Fig. 6, the presence of the magnetic field caused no significant change in the velocity distribution during the interval of measurement (30 seconds).

Some of the difficulties in the model experiment may be eliminated or greatly reduced by using liquid sodium instead of mercury, and possibly the experiment will be repeated with this change.

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